The year 2030 is many years in the future, however. Many factors could affect the tuition, and the actual figure for 2030 could turn out to be very different from our prediction.

⊞ TECHNOLOGY NOTE

You can plot data with a TI-84 Plus graphing calculator using the following steps.

1. Store the data in lists.
2. Define the stat plot.
3. Turn off Y = functions (unless you also want to graph a function).
4. Turn on the plot you want to display.
5. Define the viewing window.
6. Display the graph.

Consult the calculator's instruction booklet or the *Graphing Calculator and Excel Spreadsheet Manual*, available with this book, for specific instructions. See the calculator-generated graph in Figure 10(b), which includes the points and line from Example 14. Notice how the line closely approximates the data.

1.1 EXERCISES

Find the slope of each line.

1. Through $(4, 5)$ and $(-1, 2)$ $3/5$
2. Through $(5, -4)$ and $(1, 3)$ $-7/4$
3. Through $(8, 4)$ and $(8, -7)$ Not defined
4. Through $(1, 5)$ and $(-2, 5)$ 0
5. $y = x$ 1
6. $y = 3x - 2$ 3
7. $5x - 9y = 11$ $5/9$
8. $4x + 7y = 1$ $-4/7$
9. $x = 5$ Not defined
10. The x-axis 0
11. $y = 8$ 0
12. $y = -6$ 0
13. A line parallel to $6x - 3y = 12$ 2
14. A line perpendicular to $8x = 2y - 5$ $-1/4$

**In Exercises 15–24, find an equation in slope-intercept form for each line.**

15. Through $(1, 3), m = -2$ $y = -2x + 5$
16. Through $(2, 4), m = -1$ $y = -x + 6$
17. Through $(-5, -7), m = 0$ $y = -7$
18. Through $(-8, 1)$, with undefined slope $x = -8$
19. Through $(4, 2)$ and $(1, 3)$ $y = -(1/3)x + 10/3$
20. Through $(8, -1)$ and $(4, 3)$ $y = -x + 7$
21. Through $(2/3, 1/2)$ and $(1/4, -2)$ $y = 6x - 7/2$
22. Through $(-2, 3/4)$ and $(2/3, 5/2)$ $y = (21/32)x + 33/16$
23. Through $(-8, 4)$ and $(-8, 6)$ $x = -8$
24. Through $(-1, 3)$ and $(0, 3)$ $y = 3$

In Exercises 25–34, find an equation for each line in the form $ax + by = c$, where a, b, and c are integers with no factor common to all three and $a \geq 0$.

25. x-intercept -6, y-intercept -3 $x + 2y = -6$
26. x-intercept -2, y-intercept 4 $2x - y = -4$
27. Vertical, through $(-6, 5)$ $x = -6$
28. Horizontal, through $(8, 7)$ $y = 7$
29. Through $(-4, 6)$, parallel to $3x + 2y = 13$ $3x + 2y = 0$
30. Through $(2, -5)$, parallel to $2x - y = -4$ $2x - y = 9$
31. Through $(3, -4)$, perpendicular to $x + y = 4$ $x - y = 7$
32. Through $(-2, 6)$, perpendicular to $2x - 3y = 5$ $3x + 2y = 6$
33. The line with y-intercept 4 and perpendicular to $x + 5y = 7$ $5x - y = -4$
34. The line with x-intercept $-2/3$ and perpendicular to $2x - y = 4$ $3x + 6y = -2$
35. Do the points $(4, 3)$, $(2, 0)$, and $(-18, -12)$ lie on the same line? Explain why or why not. (*Hint:* Find the slopes between the points.) No
36. Find k so that the line through $(4, -1)$ and $(k, 2)$ is
    a. parallel to $2x + 3y = 6$, $k = -1/2$
    b. perpendicular to $5x - 2y = -1$. $k = -7/2$
37. Use slopes to show that the quadrilateral with vertices at $(1, 3)$, $(-5/2, 2)$, $(-7/2, 4)$, and $(2, 1)$ is a parallelogram.
38. Use slopes to show that the square with vertices at $(-2, 5)$, $(4, 5)$, $(4, -1)$, and $(-2, -1)$ has diagonals that are perpendicular.

For the lines in Exercises 39 and 40, which of the following is closest to the slope of the line? (a) 1 (b) 2 (c) 3 (d) 21 (e) 22 (f) −3
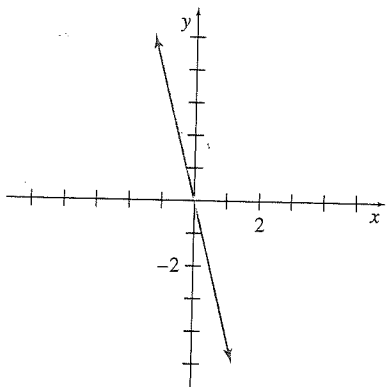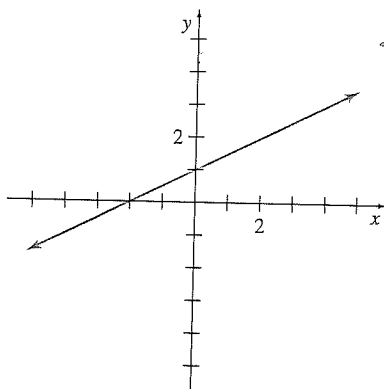
39.
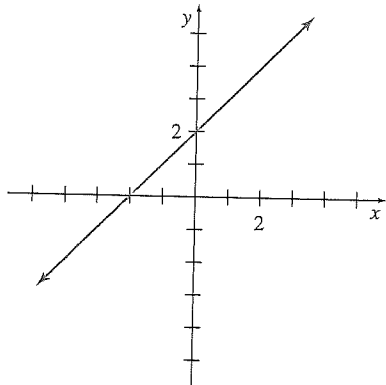
40.
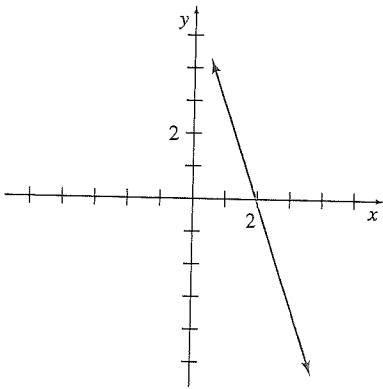
In Exercises 41 and 42, estimate the slope of the lines.

41.

42.

+ **43.** To show that two perpendicular lines, neither of which is vertical, have slopes with a product of −1, go through the following steps. Let line L_1 have equation $y = m_1 x + b_1$, and let L_2 have equation $y = m_2 x + b_2$, with $m_1 > 0$ and $m_2 < 0$. Assume that L_1 and L_2 are perpendicular, and use right triangle MPN shown in the figure. Prove each of the following statements.

a. MQ has length m_1.

b. QN has length $-m_2$.

c. Triangles MPQ and PNQ are similar.

d. $m_1/1 = 1/(-m_2)$ and $m_1 m_2 = -1$
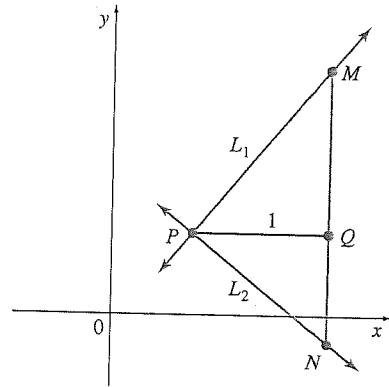
44. Consider the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

a. Show that this equation represents a line by writing it in the form $y = mx + b$. $y = -(b/a)x + b$

b. Find the x- and y-intercepts of this line. a and b

c. Explain in your own words why the equation in this exercise is known as the intercept form of a line.

Graph each equation.

45. $y = x - 1$ *

46. $y = 4x + 5$ *

47. $y = -4x + 9$ *

48. $y = -6x + 12$ *

49. $2x - 3y = 12$ *

50. $3x - y = -9$ *

51. $3y - 7x = -21$ *

52. $5y + 6x = 11$ *

53. $y = -2$ *

54. $x = 4$ *

55. $x + 5 = 0$ *

56. $y + 8 = 0$ *

57. $y = 2x$ *

58. $y = -5x$ *

59. $x + 4y = 0$ *

60. $3x - 5y = 0$ *

APPLICATIONS

Business and Economics

61. Sales The sales of a small company were $27,000 in its second year of operation and $63,000 in its fifth year. Let y represent sales in the xth year of operation. Assume that the data can be approximated by a straight line.

a. Find the slope of the sales line, and give an equation for the line in the form $y = mx + b$. 12,000; $y = 12,000x + 3000$

b. Use your answer from part a to find out how many years must pass before the sales surpass $100,000. 8 years, 1 month

62. Use of Cellular Telephones The following table shows the subscribership of cellular telephones in the United States (in millions) for even-numbered years between 2000 and 2008. *Source: Time Almanac 2010.*

Year	2000	2002	2004	2006	2008
Subscribers (in millions)	109.48	140.77	182.14	233.04	270.33

a. Plot the data by letting $t = 0$ correspond to 2000. Discuss how well the data fit a straight line. *

b. Determine a linear equation that approximates the number of subscribers using the points $(0, 109.48)$ and $(8, 270.33)$. *

c. Repeat part b using the points $(2, 140.77)$ and $(8, 270.33)$. *

d. Discuss why your answers to parts b and c are similar but not identical.

e. Using your equations from parts b and c, approximate the number of cellular phone subscribers in the year 2007. Compare your result with the actual value of 255.40 million. *

63. Consumer Price Index The Consumer Price Index (CPI) is a measure of the change in the cost of goods over time. The index was 100 for the three-year period centered on 1983. For simplicity, we will assume that the CPI was exactly 100 in 1983. Then the CPI of 215.3 in 2008 indicates that an item that cost $1.00 in 1983 would cost $2.15 in 2008. The CPI has been increasing approximately linearly over the last few decades. *Source: Time Almanac 2010.*

a. Use this information to determine an equation for the CPI in terms of t, which represents the years since 1980. $y = 4.612t + 86.164$

b. Based on the answer to part a, what was the predicted value of the CPI in 2000? Compare this estimate with the actual CPI of 172.2. 178.4, which is slightly more than the actual CPI.

c. Describe the rate at which the annual CPI is changing. *

Life Sciences

64. HIV Infection The time interval between a person's initial infection with HIV and that person's eventual development of AIDS symptoms is an important issue. The method of infection with HIV affects the time interval before AIDS develops. One study of HIV patients who were infected by intravenous drug use found that 17% of the patients had AIDS after 4 years, and 33% had developed the disease after 7 years. The relationship between the time interval and the percentage of patients with AIDS can be modeled accurately with a linear equation. *Source: Epidemiologic Review.*

a. Write a linear equation $y = mt + b$ that models this data, using the ordered pairs $(4, 0.17)$ and $(7, 0.33)$. $y = 0.053t - 0.043$

b. Use your equation from part a to predict the number of years before half of these patients will have AIDS. About 10.2 yr

+ **65. Exercise Heart Rate** To achieve the maximum benefit for the heart when exercising, your heart rate (in beats per minute) should be in the target heart rate zone. The lower limit of this zone is found by taking 70% of the difference between 220 and your age. The upper limit is found by using 85%. *Source: Physical Fitness.*

a. Find formulas for the upper and lower limits (u and l) as linear equations involving the age x. *

b. What is the target heart rate zone for a 20-year-old? *

c. What is the target heart rate zone for a 40-year-old? *

d. Two women in an aerobics class stop to take their pulse and are surprised to find that they have the same pulse. One woman is 36 years older than the other and is working at the upper limit of her target heart rate zone. The younger woman is working at the lower limit of her target heart rate zone. What are the ages of the two women, and what is their pulse? *

e. Run for 10 minutes, take your pulse, and see if it is in your target heart rate zone. (After all, this is listed as an exercise!)

+ **66. Ponies Trotting** A 1991 study found that the peak vertical force on a trotting Shetland pony increased linearly with the pony's speed, and that when the force reached a critical level, the pony switched from a trot to a gallop. For one pony, the critical force was 1.16 times its body weight. It experienced a force of 0.75 times its body weight at a speed of 2 meters per second and a force of 0.93 times its body weight at 3 meters per second. At what speed did the pony switch from a trot to a gallop? *Source: Science.* Approximately 4.3 m/sec

+ **67. Life Expectancy** Some scientists believe there is a limit to how long humans can live. One supporting argument is that during the last century, life expectancy from age 65 has increased more slowly than life expectancy from birth, so eventually these two will be equal, at which point, according to these scientists, life expectancy should increase no further. In 1900, life expectancy at birth was 46 yr, and life expectancy at age 65 was 76 yr. In 2004, these figures had risen to 77.8 and 83.7, respectively. In both cases, the increase in life expectancy has been linear. Using these assumptions and the data given, find the maximum life expectancy for humans. *Source: Science.* Approximately 86 yr

Social Sciences

68. Child Mortality Rate The mortality rate for children under 5 years of age around the world has been declining in a roughly linear fashion in recent years. The rate per 1000 live births was 90 in 1990 and 65 in 2008. *Source: World Health Organization.*

a. Determine a linear equation that approximates the mortality rate in terms of time t, where t represents the number of years since 1900. $y = -1.389t + 215$

b. If this trend continues, in what year will the mortality rate first drop to 50 or below per 1000 live births? 2019

69. Health Insurance The percentage of adults in the United States without health insurance increased at a roughly linear rate from 1999, when it was 17.2%, to 2008, when it was 20.3%. *Source: The New York Times.*

a. Determine a linear equation that approximates the percentage of adults in the United States without health insurance in terms of time t, where t represents the number of years since 1990. $y = 0.3444t + 14.1$

b. If this trend were to continue, in what year would the percentage of adults without health insurance be at least 25%? 2022

70. Marriage The following table lists the U.S. median age at first marriage for men and women. The age at which both groups marry for the first time seems to be increasing at a roughly linear rate in recent decades. Let t correspond to the number of years since 1980. *Source: U.S. Census Bureau.*

Age at First Marriage						
Year	1980	1985	1990	1995	2000	2005
Men	24.7	25.5	26.1	26.9	26.8	27.1
Women	22.0	23.3	23.9	24.5	25.1	25.3

a. Find a linear equation that approximates the data for men, using the data for the years 1980 and 2005. $y = 0.096t + 24.7$

b. Repeat part a using the data for women. $y = 0.132t + 22.0$

c. Which group seems to have the faster increase in median age at first marriage? Women

d. In what year will the men's median age at first marriage reach 30? 2036 (or 2035, depending on how you round)

e. When the men's median age at first marriage is 30, what will the median age be for women? 29.4 (or 29.3 if 2035 is used)

71. **Immigration**　In 1950, there were 249,187 immigrants admitted to the United States. In 2008, the number was 1,107,126. *Source: 2008 Yearbook of Immigration Statistics.*

a. Assuming that the change in immigration is linear, write an equation expressing the number of immigrants, y, in terms of t, the number of years after 1900. $y = 14,792.05t - 490,416$

b. Use your result in part a to predict the number of immigrants admitted to the United States in 2015. 1,210,670

c. Considering the value of the y-intercept in your answer to part a, discuss the validity of using this equation to model the number of immigrants throughout the entire 20th century.

Physical Sciences

72. **Global Warming**　In 1990, the Intergovernmental Panel on Climate Change predicted that the average temperature on Earth would rise 0.3°C per decade in the absence of international controls on greenhouse emissions. Let t measure the time in years since 1970, when the average global temperature was 15°C. *Source: Science News.*

a. Find a linear equation giving the average global temperature in degrees Celsius in terms of t, the number of years since 1970. $T = 0.03t + 15$

b. Scientists have estimated that the sea level will rise by 65 cm if the average global temperature rises to 19°C. According to your answer to part a, when would this occur? About 2103

73. **Galactic Distance**　The table lists the distances (in megaparsecs where 1 megaparsec $\approx 3.1 \times 10^{19}$ km) and velocities (in kilometers per second) of four galaxies moving rapidly away from Earth. *Source: Astronomical Methods and Calculations, and Fundamental Astronomy.*

Galaxy	Distance	Velocity
Virga	15	1600
Ursa Minor	200	15,000
Corona Borealis	290	24,000
Bootes	520	40,000

a. Plot the data points letting x represent distance and y represent velocity. Do the points lie in an approximately linear pattern? There appears to be a linear relationship.

b. Write a linear equation $y = mx$ to model this data, using the ordered pair (520, 40,000). $y = 76.9x$

c. The galaxy Hydra has a velocity of 60,000 km per sec. Use your equation to approximate how far away it is from Earth. About 780 megaparsecs (about 1.5×10^{22} mi)

d. The value of m in the equation is called the *Hubble constant.* The Hubble constant can be used to estimate the age of the universe A (in years) using the formula

$$A = \frac{9.5 \times 10^{11}}{m}.$$

Approximate A using your value of m. About 12.4 billion yr

General Interest

74. **News/Talk Radio**　From 2001 to 2007, the number of stations carrying news/talk radio increased at a roughly linear rate, from 1139 in 2001 to 1370 in 2007. *Source: State of the Media.*

a. Find a linear equation expressing the number of stations carrying news/talk radio, y, in terms of t, the years since 2000. $y = 38.5t + 1100.5$

b. Use your answer from part a to predict the number of stations carrying news/talk radio in 2008. Compare with the actual number of 2046. Discuss how the linear trend from 2001 to 2007 might have changed in 2008. 1408 or 1409

75. **Tuition**　The table lists the annual cost (in dollars) of tuition and fees at private four-year colleges for selected years. (See Example 14.) *Source: The College Board.*

Year	Tuition and Fees
2000	16,072
2002	18,060
2004	20,045
2006	22,308
2008	25,177
2009	26,273

a. Sketch a graph of the data. Do the data appear to lie roughly along a straight line? *

b. Let $t = 0$ correspond to the year 2000. Use the points (0, 16,072) and (9, 26,273) to determine a linear equation that models the data. What does the slope of the graph of the equation indicate? *

c. Discuss the accuracy of using this equation to estimate the cost of private college in 2025. The year 2025 is too far in the future to rely on this equation to predict costs; too many other factors may influence these costs by then.

YOUR TURN ANSWERS

1. 1/3

2. $y = (3/2)x + 6$

3. $-8/3$

4. $y = -2x + 13$

5. $y = (1/2)x + 3$

6. $y = (3/2)x - 5/2$

The F-intercept of the graph is 32, so by the slope-intercept form, the equation of the line is

$$F = \frac{9}{5}C + 32.$$

With simple algebra this equation can be rewritten to give C in terms of F:

$$C = \frac{5}{9}(F - 32).$$

1.2   EXERCISES

For Exercises 1–10, let $f(x) = 7 - 5x$ and $g(x) = 2x - 3$. Find the following.

1. $f(2)$ -3

2. $f(4)$ -13

3. $f(-3)$ 22

4. $f(-1)$ 12

5. $g(1.5)$ 0

6. $g(2.5)$ 2

7. $g(-1/2)$ -4

8. $g(-3/4)$ $-9/2$

9. $f(t)$ $7 - 5t$

10. $g(k^2)$ $2k^2 - 3$

In Exercises 11–14, decide whether the statement is true or false.

11. To find the x-intercept of the graph of a linear function, we solve $y = f(x) = 0$, and to find the y-intercept, we evaluate $f(0)$. True

12. The graph of $f(x) = -5$ is a vertical line. False

13. The slope of the graph of a linear function cannot be undefined. True

14. The graph of $f(x) = ax$ is a straight line that passes through the origin. True

15. Describe what fixed costs and marginal costs mean to a company.

16. In a few sentences, explain why the price of a commodity not already at its equilibrium price should move in that direction.

17. Explain why a linear function may not be adequate for describing the supply and demand functions.

18. In your own words, describe the break-even quantity, how to find it, and what it indicates.

Write a linear cost function for each situation. Identify all variables used.

19. A Lake Tahoe resort charges a snowboard rental fee of $10 plus $2.25 per hour. *

20. An Internet site for downloading music charges a $10 registration fee plus 99 cents per downloaded song. *

21. A parking garage charges 2 dollars plus 75 cents per half-hour. *

22. For a one-day rental, a car rental firm charges $44 plus 28 cents per mile. *

* indicates answer is in the Additional Instructor Answers at end of the book.

Assume that each situation can be expressed as a linear cost function. Find the cost function in each case.

23. Fixed cost: $100; 50 items cost $1600 to produce.
$C(x) = 30x + 100$

24. Fixed cost: $35; 8 items cost $395 to produce.
$C(x) = 45x + 35$

25. Marginal cost: $75; 50 items cost $4300 to produce.
$C(x) = 75x + 550$

26. Marginal cost: $120; 700 items cost $96,500 to produce.
$C(x) = 120x + 12,500$

APPLICATIONS

Business and Economics

27. Supply and Demand   Suppose that the demand and price for a certain model of a youth wristwatch are related by

$$p = D(q) = 16 - 1.25q,$$

where p is the price (in dollars) and q is the quantity demanded (in hundreds). Find the price at each level of demand.

a. 0 watches $16   **b.** 400 watches $11   **c.** 800 watches $6

Find the quantity demanded for the watch at each price.

d. $8 640 watches   **e.** $10 480 watches   **f.** $12 320 watches

g. Graph $p = 16 - 1.25q$. *

Suppose the price and supply of the watch are related by

$$p = S(q) = 0.75q,$$

where p is the price (in dollars) and q is the quantity supplied (in hundreds) of watches. Find the quantity supplied at each price.

h. $0 0 watches   **i.** $10 watches About 1333   **j.** $20 watches About 2667

k. Graph $p = 0.75q$ on the same axis used for part g. *

l. Find the equilibrium quantity and the equilibrium price.
800 watches, $6

28. Supply and Demand   Suppose that the demand and price for strawberries are related by

$$p = D(q) = 5 - 0.25q,$$

where p is the price (in dollars) and q is the quantity demanded (in hundreds of quarts). Find the price at each level of demand.

a. 0 quarts $5   **b.** 400 quarts $4   **c.** 840 quarts $2.90

Find the quantity demanded for the strawberries at each price.

d. $4.50 200 quarts   **e.** $3.25 700 quarts   **f.** $2.40 1040 quarts

g. Graph $p = 5 - 0.25q$. *

Suppose the price and supply of strawberries are related by

$$p = S(q) = 0.25q,$$

where p is the price (in dollars) and q is the quantity supplied (in hundreds of quarts) of strawberries. Find the quantity supplied at each price.

h. $0 0 quarts   **i.** $2 800 quarts   **j.** $4.50 1800 quarts

k. Graph $p = 0.75q$ on the same axis used for part g. *

l. Find the equilibrium quantity and the equilibrium price.
1000 quarts, $2.50

29. Supply and Demand Let the supply and demand functions for butter pecan ice cream be given by

$$p = S(q) = \frac{2}{5}q \quad \text{and} \quad p = D(q) = 100 - \frac{2}{5}q,$$

where p is the price in dollars and q is the number of 10-gallon tubs.

a. Graph these on the same axes. *

b. Find the equilibrium quantity and the equilibrium price. (*Hint:* The way to divide by a fraction is to multiply by its reciprocal.) 125 tubs, $50

30. Supply and Demand Let the supply and demand functions for sugar be given by

$$p = S(q) = 1.4q - 0.6 \quad \text{and}$$
$$p = D(q) = -2q + 3.2,$$

where p is the price per pound and q is the quantity in thousands of pounds.

a. Graph these on the same axes. *

b. Find the equilibrium quantity and the equilibrium price.
About 1120 lb; about $0.96

31. Supply and Demand Suppose that the supply function for honey is $p = S(q) = 0.3q + 2.7$, where p is the price in dollars for an 8-oz container and q is the quantity in barrels. Suppose also that the equilibrium price is $4.50 and the demand is 2 barrels when the price is $6.10. Find an equation for the demand function, assuming it is linear. $D(q) = 6.9 - 0.4q$

32. Supply and Demand Suppose that the supply function for walnuts is $p = S(q) = 0.25q + 3.6$, where p is the price in dollars per pound and q is the quantity in bushels. Suppose also that the equilibrium price is $5.85, and the demand is 4 bushels when the price is $7.60. Find an equation for the demand function, assuming it is linear. $D(q) = 9 - 0.35q$

33. T-Shirt Cost Joanne Wendelken sells silk-screened T-shirts at community festivals and crafts fairs. Her marginal cost to produce one T-shirt is $3.50. Her total cost to produce 60 T-shirts is $300, and she sells them for $9 each.

a. Find the linear cost function for Joanne's T-shirt production.
$C(x) = 3.50x + 90$

b. How many T-shirts must she produce and sell in order to break even? 17 shirts

c. How many T-shirts must she produce and sell to make a profit of $500? 108 shirts

34. Publishing Costs Alfred Juarez owns a small publishing house specializing in Latin American poetry. His fixed cost to produce a typical poetry volume is $525, and his total cost to produce 1000 copies of the book is $2675. His books sell for $4.95 each.

a. Find the linear cost function for Alfred's book production.
$C(x) = 2.15x + 525$

b. How many poetry books must he produce and sell in order to break even? 188

c. How many books must he produce and sell to make a profit of $1000? 545

35. Marginal Cost of Coffee The manager of a restaurant found that the cost to produce 100 cups of coffee is $11.02, while the cost to produce 400 cups is $40.12. Assume the cost $C(x)$ is a linear function of x, the number of cups produced.

a. Find a formula for $C(x)$. $C(x) = 0.097x + 1.32$

b. What is the fixed cost? $1.32

c. Find the total cost of producing 1000 cups. $98.32

d. Find the total cost of producing 1001 cups. $98.42

e. Find the marginal cost of the 1001st cup. 9.7 ¢

f. What is the marginal cost of *any* cup and what does this mean to the manager? *

36. Marginal Cost of a New Plant In deciding whether to set up a new manufacturing plant, company analysts have decided that a linear function is a reasonable estimation for the total cost $C(x)$ in dollars to produce x items. They estimate the cost to produce 10,000 items as $547,500, and the cost to produce 50,000 items as $737,500.

a. Find a formula for $C(x)$. $C(x) = 500,000 + 4.75x$

b. Find the fixed cost. $500,000

c. Find the total cost to produce 100,000 items. $975,000

d. Find the marginal cost of the items to be produced in this plant and what does this mean to the manager? *

37. Break-Even Analysis Producing x units of tacos costs $C(x) = 5x + 20$; revenue is $R(x) = 15x$, where $C(x)$ and $R(x)$ are in dollars.

a. What is the break-even quantity? 2 units

b. What is the profit from 100 units? $980

c. How many units will produce a profit of $500? 52 units

38. Break-Even Analysis To produce x units of a religious medal costs $C(x) = 12x + 39$. The revenue is $R(x) = 25x$. Both $C(x)$ and $R(x)$ are in dollars.

a. Find the break-even quantity. 3 units

b. Find the profit from 250 units. $3211

c. Find the number of units that must be produced for a profit of $130. 13 units

Break-Even Analysis You are the manager of a firm. You are considering the manufacture of a new product, so you ask the accounting department for cost estimates and the sales department for sales estimates. After you receive the data, you must decide whether to go ahead with production of the new product. Analyze the data in Exercises 39–42 (find a break-even

quantity) and then decide what you would do in each case. Also write the profit function.

39. $C(x) = 85x + 900$; $R(x) = 105x$; no more than 38 units can be sold. *

40. $C(x) = 105x + 6000$; $R(x) = 250x$; no more than 400 units can be sold. *

41. $C(x) = 70x + 500$; $R(x) = 60x$ (*Hint:* What does a negative break-even quantity mean?) *

42. $C(x) = 1000x + 5000$; $R(x) = 900x$ *

43. Break-Even Analysis Suppose that the fixed cost for a product is $400 and the break-even quantity is 80. Find the marginal profit (the slope of the linear profit function). 5

44. Break-Even Analysis Suppose that the fixed cost for a product is $650 and the break-even quantity is 25. Find the marginal profit (the slope of the linear profit function). 26

Physical Sciences

45. Temperature Use the formula for conversion between Fahrenheit and Celsius derived in Example 7 to convert each temperature.

a. 58°F to Celsius 14.4°C

b. −20°F to Celsius −28.9°C

c. 50°C to Fahrenheit 122°F

46. Body Temperature You may have heard that the average temperature of the human body is 98.6°. Recent experiments show that the actual figure is closer to 98.2°. The figure of 98.6 comes from experiments done by Carl Wunderlich in 1868. But Wunderlich measured the temperatures in degrees Celsius and rounded the average to the nearest degree, giving 37°C as the average temperature. *Source: Science News.*

a. What is the Fahrenheit equivalent of 37°C? 98.6°F

b. Given that Wunderlich rounded to the nearest degree Celsius, his experiments tell us that the actual average human body temperature is somewhere between 36.5°C and 37.5°C. Find what this range corresponds to in degrees Fahrenheit. 97.7°F and 99.5°F

47. Temperature Find the temperature at which the Celsius and Fahrenheit temperatures are numerically equal. −40°

General Interest

48. Education Cost The 2009–2010 budget for the California State University system projected a fixed cost of $486,000 at each of five off-campus centers, plus a marginal cost of $1140 per student. *Source: California State University.*

a. Find a formula for the cost at each center, $C(x)$, as a linear function of x, the number of students. $C(x) = 1140x + 486,000$

b. The budget projected 500 students at each center. Calculate the total cost at each center. $1,056,000

c. Suppose, due to budget cuts, that each center is limited to $1 million. What is the maximum number of students that each center can then support? 450

YOUR TURN ANSWERS

1. 25 **2.** 7600 and 4400

3. 8000 watermelons and $3.20 per watermelon

4. $C(x) = 15x + 730$ **5.** 360

1.3 The Least Squares Line

How has the accidental death rate in the United States changed over time?
In Example 1 in this section, we show how to answer such questions using the method of least squares.

We use past data to find trends and to make tentative predictions about the future. The only assumption we make is that the data are related linearly—that is, if we plot pairs of data, the resulting points will lie close to some line. This method cannot give exact answers. The best we can expect is that, if we are careful, we will get a reasonable approximation.

The table lists the number of accidental deaths per 100,000 people in the United States through the past century. *Source: National Center for Health Statistics.* If you were a manager at an insurance company, these data could be very important. You might need to make some predictions about how much you will pay out next year in accidental death benefits, and even a very tentative prediction based on past trends is better than no prediction at all.

The first step is to draw a scatterplot, as we have done in Figure 15 on the next page. Notice that the points lie approximately along a line, which means that a linear function may give a good approximation of the data. If we select two points and find the line that passes through them, as we did in Section 1.1, we will get a different line for each pair of points, and in some cases the lines will be very different. We want to draw one line that is simultaneously close to all the points on the graph, but many such lines are possible, depending upon how we define the phrase "simultaneously close to all the points." How do we decide on the best possible line? Before going on, you might want to try drawing the line you think is best on Figure 15.

Accidental Death Rate	
Year	Death Rate
1910	84.4
1920	71.2
1930	80.5
1940	73.4
1950	60.3
1960	52.1
1970	56.2
1980	46.5
1990	36.9
2000	34.0

SOLUTION   For w in the interval $(0, 2]$, the shipping cost is $y = 25$. For w in $(2, 3]$, the shipping cost is $y = 25 + 3 = 28$. For w in $(3, 4]$, the shipping cost is $y = 28 + 3 = 31$, and so on. The graph is shown in Figure 11.

The function discussed in Example 7 is called a **step function**. Many real-life situations are best modeled by step functions. Additional examples are given in the exercises.

In Chapter 1 you saw several examples of linear models. In Example 8, we use a quadratic equation to model the area of a lot.

EXAMPLE 8   Area

A fence is to be built against a brick wall to form a rectangular lot, as shown in Figure 12. Only three sides of the fence need to be built, because the wall forms the fourth side. The contractor will use 200 m of fencing. Let the length of the wall be l and the width w, as shown in Figure 12.
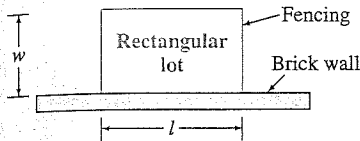
FIGURE 12

(a) Find the area of the lot as a function of the length l.

SOLUTION   The area formula for a rectangle is area = length × width, or

$$A = lw.$$

We want the area as a function of the length only, so we must eliminate the width. We use the fact that the total amount of fencing is the sum of the three sections, one length and two widths, so $200 = l + 2w$. Solve this for w:

$$200 = l + 2w$$
$$200 - l = 2w \qquad \text{Subtract } l \text{ from both sides.}$$
$$100 - l/2 = w. \qquad \text{Divide both sides by 2.}$$

Substituting this into the formula for area gives

$$A = l(100 - l/2).$$

(b) Find the domain of the function in part (a).

SOLUTION   The length cannot be negative, so $l \geq 0$. Similarly, the width cannot be negative, so $100 - l/2 \geq 0$, from which we find $l \leq 200$. Therefore, the domain is $[0, 200]$.

(c) Sketch a graph of the function in part (a).

FIGURE 13

SOLUTION   The result from a graphing calculator is shown in Figure 13. Notice that at the endpoints of the domain, when $l = 0$ and $l = 200$, the area is 0. This makes sense: If the length or width is 0, the area will be 0 as well. In between, as the length increases from 0 to 100 m, the area gets larger, and seems to reach a peak of 5000 m^2 when $l = 100$ m. After that, the area gets smaller as the length continues to increase because the width is becoming smaller.

In the next section, we will study this type of function in more detail and determine exactly where the maximum occurs.
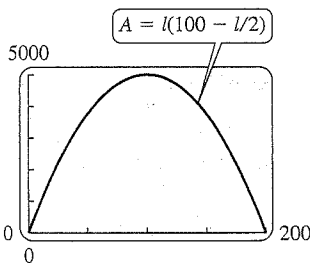
2.1   EXERCISES

Which of the following rules define y as a function of x?

1.

Not a function

2.

Function

3.

x	y
3	9
2	4
1	1
0	0
−1	1
−2	4
−3	9

Function

4.

x	y
9	3
4	2
1	1
0	0
1	−1
4	−2
9	−3

Not a function

5. $y = x^3 + 2$ Function

6. $y = \sqrt{x}$ Function

7. $x = |y|$ Not a function

8. $x = y^2 + 4$ Not a function

List the ordered pairs obtained from each equation, given {−2, −1, 0, 1, 2, 3} as the domain. Graph each set of ordered pairs. Give the range.

9. $y = 2x + 3$ *

10. $y = -3x + 9$ *

11. $2y - x = 5$ *

12. $6x - y = -1$ *

13. $y = x(x + 2)$ *

14. $y = (x - 2)(x + 2)$ *

15. $y = x^2$ *

16. $y = -4x^2$ *

Give the domain of each function defined as follows.

17. $f(x) = 2x$ $(-\infty, \infty)$

18. $f(x) = 2x + 3$ $(-\infty, \infty)$

19. $f(x) = x^4$ $(-\infty, \infty)$

20. $f(x) = (x + 3)^2$ $(-\infty, \infty)$

21. $f(x) = \sqrt{4 - x^2}$ $[-2, 2]$

22. $f(x) = |3x - 6|$ $(-\infty, \infty)$

23. $f(x) = (x - 3)^{1/2}$ $[3, \infty)$

24. $f(x) = (3x + 5)^{1/2}$ $[-5/3, \infty)$

25. $f(x) = \dfrac{2}{1 - x^2}$ *

26. $f(x) = \dfrac{-8}{x^2 - 36}$ *

27. $f(x) = -\sqrt{\dfrac{2}{x^2 - 16}}$ *

28. $f(x) = -\sqrt{\dfrac{5}{x^2 + 36}}$ $(-\infty, \infty)$

29. $f(x) = \sqrt{x^2 - 4x - 5}$ *

30. $f(x) = \sqrt{15x^2 + x - 2}$ $(-\infty, -2/5] \cup [1/3, \infty)$

31. $f(x) = \dfrac{1}{\sqrt{3x^2 + 2x - 1}}$ $(-\infty, -1) \cup (1/3, \infty)$

32. $f(x) = \sqrt{\dfrac{x^2}{3 - x}}$ $(-\infty, 3)$

Give the domain and the range of each function. Where arrows are drawn, assume the function continues in the indicated direction.
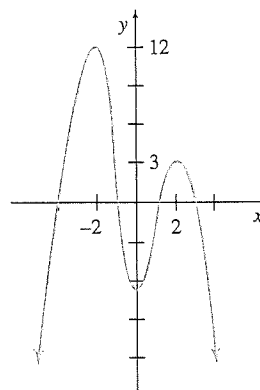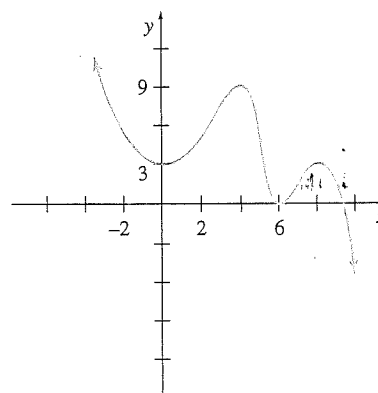
33.
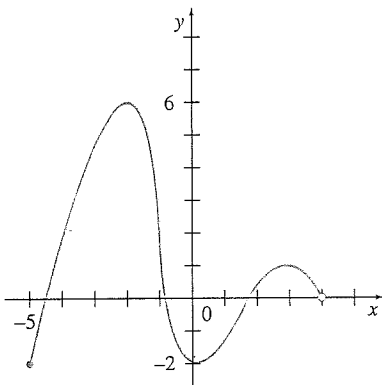
Domain: $[-5, 4)$; range: $[-2, 6]$

34.
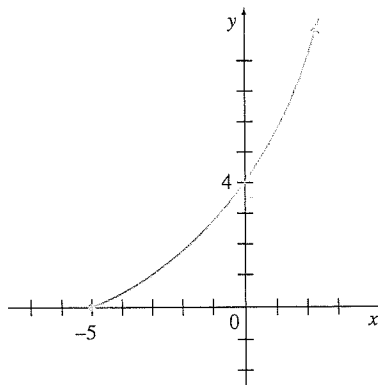
Domain: $[-5, \infty)$; range: $[0, \infty)$

35.

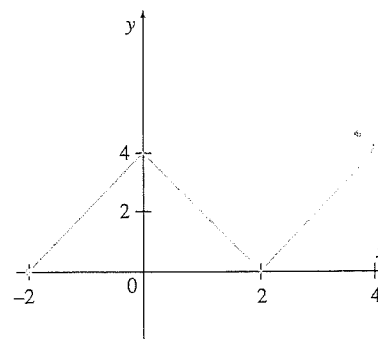Domain: $(-\infty, \infty)$; range: $(-\infty, 12]$

36.

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$
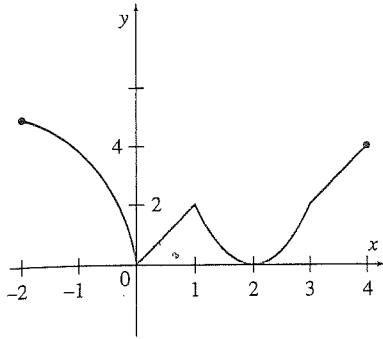
In Exercises 37–40, give the domain and range. Then, use each graph to find (a) $f(-2)$, (b) $f(0)$, (c) $f(1/2)$, and (d) any values of x such that $f(x) = 1$.
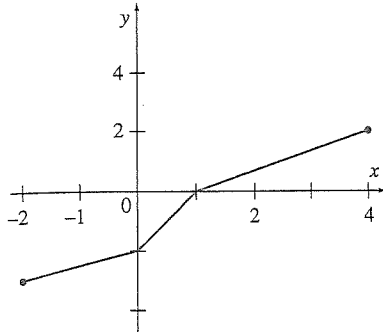
37.

Domain: $[-2, 4]$; range: $[0, 4]$
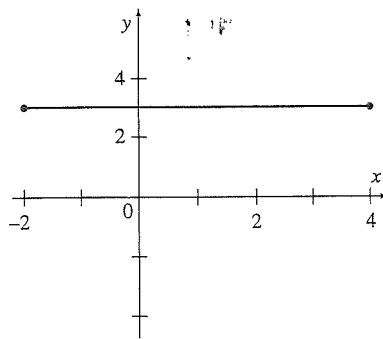a. 0 b. 4 c. 3
d. −1.5, 1.5, 2.5

38.

Domain: [−2, 4];
range: [0, 5]
a. 5 b. 0 c. 1
d. −0.2, 0.5, 1.2, 2.8

39.

Domain: [−2, 4];
range: [−3, 2]
a. −3 b. −2
c. −1 d. 2.5

40.

Domain: [−2, 4];
range: {3}
a. 3 b. 3 c. 3
d. Nowhere

For each function, find (a) $f(4)$, (b) $f(-1/2)$, (c) $f(a)$, (d) $f(2/m)$, and (e) any values of x such that $f(x) = 1$.

41. $f(x) = 3x^2 - 4x + 1$ *

42. $f(x) = (x + 3)(x - 4)$ *

43. $f(x) = \begin{cases} \dfrac{2x + 1}{x - 4} & \text{if } x \neq 4 \\ 7 & \text{if } x = 4 \end{cases}$

a. 7 b. 0
c. $(2a + 1)/(a - 4)$ if $a \neq 4$,
7 if $a = 4$ d. $(4 + m)/(2 - 4m)$
if $m \neq 1/2$, 7 if $m = 1/2$ e. −5

44. $f(x) = \begin{cases} \dfrac{x - 4}{2x + 1} & \text{if } x \neq -\dfrac{1}{2} \\ 10 & \text{if } x = -\dfrac{1}{2} \end{cases}$

a. 0 b. 10
c. $(a - 4)/(2a + 1)$ if
$a \neq -1/2$, 10 if $a = -1/2$
d. $(2 - 4m)/(4 + m)$ if
$m \neq -4$, 10 if $m = -4$ e. −5

Let $f(x) = 6x^2 - 2$ and $g(x) = x^2 - 2x + 5$ to find the following values.

45. $f(t + 1)$ $6t^2 + 12t + 4$

46. $f(2 - r)$ $6r^2 - 24r + 22$

47. $g(r + h)$ *

48. $g(z - p)$ *

49. $g\left(\dfrac{3}{q}\right)$ $9/q^2 - 6/q + 5$ or $(9 - 6q + 5q^2)/q^2$

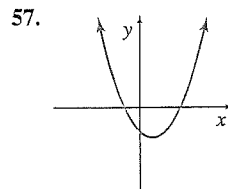**50.** $g\left(-\dfrac{5}{z}\right)$ $25/z^2 + 10/z + 5$ or $(25 + 10z + 5z^2)/z^2$

For each function defined as follows, find (a) $f(x + h)$, (b) $f(x + h) - f(x)$, and (c) $[f(x + h) - f(x)]/h$.

51. $f(x) = 2x + 1$
a. $2x + 2h + 1$ b. $2h$ c. 2

52. $f(x) = x^2 - 3$
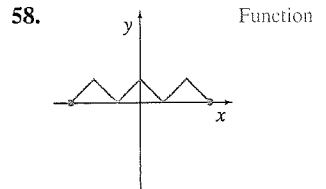a. $x^2 + 2xh + h^2 - 3$ b. $2xh + h^2$
c. $2x + h$

53. $f(x) = 2x^2 - 4x - 5$ *

54. $f(x) = -4x^2 + 3x + 2$ *

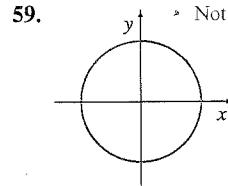**55.** $f(x) = \dfrac{1}{x}$ *

56. $f(x) = -\dfrac{1}{x^2}$ *
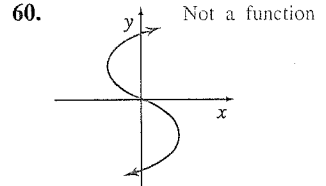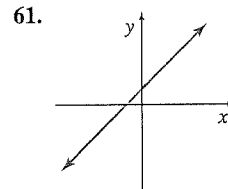
Decide whether each graph represents a function.

57.

Function

58.

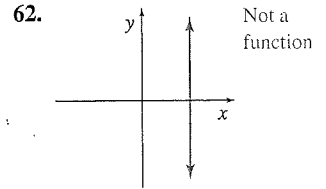Function

59.

Not a function

60.

Not a function

61.

Function

62.

Not a function

Classify each of the functions in Exercises 63–70 as even, odd, or neither.

63. $f(x) = 3x$ Odd

64. $f(x) = 5x$ Odd

65. $f(x) = 2x^2$ Even

66. $f(x) = x^2 - 3$ Even

67. $f(x) = \dfrac{1}{x^2 + 4}$ Even

68. $f(x) = x^3 + x$ Odd

69. $f(x) = \dfrac{x}{x^2 - 9}$ Odd

70. $f(x) = |x - 2|$ Neither

APPLICATIONS

Business and Economics

71. Saw Rental A chain-saw rental firm charges $28 per day or fraction of a day to rent a saw, plus a fixed fee of $8 for resharpening the blade. Let $S(x)$ represent the cost of renting a saw for x days. Find the following.

a. $S\left(\dfrac{1}{2}\right)$ $36 **b.** $S(1)$ $36 **c.** $S\left(1\dfrac{1}{4}\right)$ $64

d. $S\left(3\dfrac{1}{2}\right)$ $120 **e.** $S(4)$ $120 **f.** $S\left(4\dfrac{1}{10}\right)$ $148

g. What does it cost to rent a saw for $4\frac{9}{10}$ days? $148

h. A portion of the graph of $y = S(x)$ is shown here. Explain how the graph could be continued.
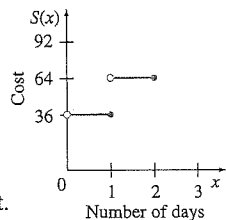
i. What is the independent variable?
   x, the number of full and partial days

j. What is the dependent variable?
   S, the cost of renting a saw

k. Is S a linear function? Explain.

l. Write a sentence or two explaining what part f and its answer represent.

m. We have left $x = 0$ out of the graph. Discuss why it should or shouldn't be included. If it were included, how would you define $S(0)$?

72. Rental Car Cost The cost to rent a mid-size car is $54 per day or fraction of a day. If the car is picked up in Pittsburgh and dropped off in Cleveland, there is a fixed $44 drop-off charge. Let $C(x)$ represent the cost of renting the car for x days, taking it from Pittsburgh to Cleveland. Find the following.

a. $C(3/4)$ $98 **b.** $C(9/10)$ $98 **c.** $C(1)$ $98 **d.** $C\left(1\dfrac{5}{8}\right)$ $152

e. Find the cost of renting the car for 2.4 days. $206

f. Graph $y = C(x)$. *

g. Is C a function? Explain. Yes

h. Is C a linear function? Explain. No

i. What is the independent variable?
x, the number of full and partial days
j. What is the dependent variable? C, the cost of renting the car

73. Attorney Fees According to Massachusetts state law, the maximum amount of a jury award that attorneys can receive is:

40% of the first $150,000,

33.3% of the next $150,000,

30% of the next $200,000, and

24% of anything over $500,000.

Let $f(x)$ represent the maximum amount of money that an attorney in Massachusetts can receive for a jury award of size x. Find each of the following, and describe in a sentence what the answer tells you. *Source: The New Yorker.*

a. $f(250,000)$ * **b.** $f(350,000)$ * **c.** $f(550,000)$ *

d. Sketch a graph of $f(x)$. *

74. Tax Rates In New York state in 2010, the income tax rates for a single person were as follows:

4% of the first $8000 earned,

4.5% of the next $3000 earned,

5.25% of the next $2000 earned,

5.9% of the next $7000 earned,

6.85% of the next $180,000 earned,

7.85% of the next $300,000 earned, and

8.97% of any amount earned over $500,000.

Let $f(x)$ represent the amount of tax owed on an income of x dollars. Find each of the following, and explain in a sentence what the answer tells you. *Source: New York State.*

a. $f(10,000)$ $410. A tax of $410 was due on an income of $10,000.

**b.** $f(12,000)$ $507.50. A tax of $507.50 was due on an income of $12,000.

c. $f(18,000)$ $855. A tax of $855 was due on an income of $18,000.

d. Sketch a graph of $f(x)$. *

Life Sciences

75. Whales Diving The figure in the next column shows the depth of a diving sperm whale as a function of time, as recorded by researchers at the Woods Hole Oceanographic Institution in Massachusetts. *Source: Peter Tyack, Woods Hole Oceanographic Institution.*
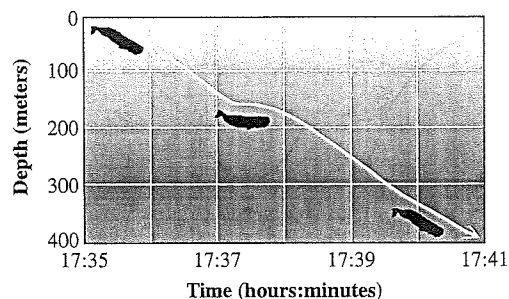
Find the depth of the whale at the following times.

a. 17 hours and 37 minutes About 140 m

b. 17 hours and 39 minutes About 250 m

76. Metabolic Rate The basal metabolic rate (in kcal/day) for large anteaters is given by

$$y = f(x) = 19.7x^{0.753},$$

where x is the anteater's weight in kilograms.* *Source: Wildlife Feeding and Nutrition.*

a. Find the basal metabolic rate for anteaters with the following weights.

i. 5 kg 66 kcal/day **ii.** 25 kg 222 kcal/day

b. Suppose the anteater's weight is given in pounds rather than kilograms. Given that 1 lb = 0.454 kg, find a function $x = g(z)$ giving the anteater's weight in kilograms if z is the animal's weight in pounds. $x = g(z) = 0.454z$

c. Write the basal metabolic rate as a function of the weight in pounds in the form $y = az^b$ by calculating $f(g(z))$. $y = 10.9z^{0.753}$

77. Swimming Energy The energy expenditure (in kcal/km) for animals swimming at the surface of the water is given by

$$y = f(x) = 0.01x^{0.88},$$

where x is the animal's weight in grams. *Source: Wildlife Feeding and Nutrition.*

a. Find the energy for the following animals swimming at the surface of the water.

i. A muskrat weighing 800 g 3.6 kcal/km

ii. A sea otter weighing 20,000 g 61 kcal/km

b. Suppose the animal's weight is given in kilograms rather than grams. Given that 1 kg = 1000 g, find a function $x = g(z)$ giving the animal's weight in grams if z is the animal's weight in kilograms. $x = g(z) = 1000z$

c. Write the energy expenditure as a function of the weight in kilograms in the form $y = az^b$ by calculating $f(g(z))$. $y = 4.4z^{0.88}$

*Technically, kilograms are a measure of mass, not weight. Weight is a measure of the force of gravity, which varies with the distance from the center of Earth. For objects on the surface of Earth, weight and mass are often used interchangeably, and we will do so in this text.